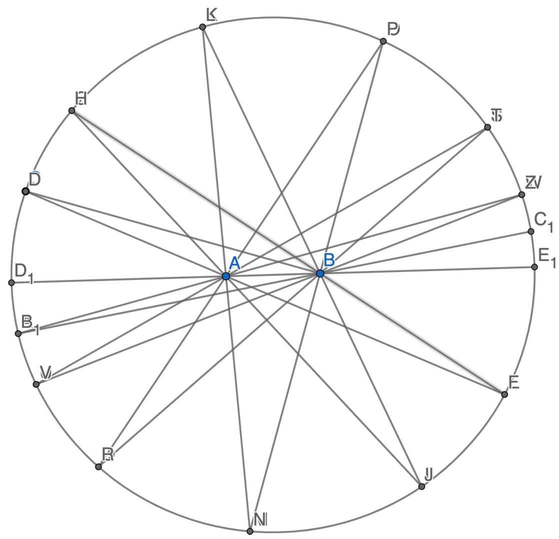
LIGHT RAY BOUNCING IN AN ELLIPSOID OF REVOLUTION

MARSHALL BUCK



Imagine that the inside surface of an ellipsoid having foci A and B is a perfect mirror. The line AB connecting the foci we call the *diameter*. (We assume that the ellipsoid is rotationally symmetric about the diameter axis.) Suppose that there is a small hole in the mirror where the ray $\overrightarrow{BE_1}$ meets the surface. Show that any light ray starting at either focus will eventually exit the ellipsoid!

In the figure above, a ray starts at B (at time 0) and heads towards D, bouncing there so that it goes through A on its way to E, where is bounces back through B. Notice that the angle of the ray leaving B each time rotates to the right. Show that the angle between the ray leaving B and the diameter becomes arbitrary small!

Extra credit 1: Express the sequence of angles algebraically, and show that the convergence to 0 is in some sense geometric.

Extra credit 2: Can you say something about the times at which a ray can exit? For example, do the possible exit times form (approximately) an arithmetic sequence, or only the exit times when the ray exits after most recently visiting B?

Extra credit 3: If an explosion of light emanates at time $t = 0$ from the focus B, with the energy uniformly distributed by direction, how is the energy exiting the ellipsoid distributed over time? What is the distance in space between the pulses of light exiting the enclosure?

Remarks on Solutions

(1) If the light ray leaving point B the kth time hits the ellipse at P_k, and we define $\theta_k := \cot(\angle BE_1P_k)$, then there is a $\lambda < 1$ such that $\theta_k = \lambda^{k-1}\theta_1$ for all $k \geq 1$.

(2) If the length of the diameter is d and the distance between the foci is f, then the length of any light path from B to A, bouncing once on the mirror, is d. The direct distance from B to the exit is $(d - f)/2$. If the ray starts heading almost directly towards A, it will come back through A and exit after distance $(3d + f)/2$. Otherwise, exit distances are of the form $(d - f)/2 + 2kd$ for $k \in \mathbb{N}$.

(3) The distance between pulses will be $2d$. The energy of pulse will at first increase geometrically, level out, and decrease geometrically by the inverse ratio to the initial increase ratio, until the pulses abruptly stop.

(4) $\lambda = ((d - f/2)/(d + f/2))^2$.